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A REVIEW ON RAINBOW RAMSEY NUMBERS WITH MULTIPLE THEOREMS APPLIED

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ABSTRACT

This paper presents an overview of the current state in research directions in the rainbow Ramsey theory. We list results, problems, andconjectures related to the existence of rainbow arithmetic progressions in [n] and N. A general perspective on other rainbow Ramsey type problems is given.

INTRODUCTION

Ramsey theory can be described as the study of unavoidable regularity in large structures. In the words of T. Motzkin, "complete disorder is impossible" [16]. In [22], we started a new trend, which can be categorized as rainbow Ramsey theory. We are interested in the existence of rainbow/hetero-chromatic structures in a colored universe, under certain density conditions on the coloring. The general goal is to show that complete disorder is unavoidable as well.Previous work regarding the existence off rainbow structures in a colored universe has been done in the context of canonical Ramsey theory (see [11, 10, 9, 33, 31, 25, 27, 26, 35] and references therein). However, the canonical theorems prove the existence of either a monochromatic structure or a rainbow structure. The results obtained in [22, 23, 5, 14] are not "either-or"–type statements and, thus, are thefirst results in the literature guaranteeing solely the existence of rainbow structures in colored sets of integers. In a sense, the conjectures and theorems we describe below can be thought of as the first rainbow counterparts of classical

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theorems in Ramsey theory, such as van der Waerden's, Rado's and Szemer´edi's theorems [44, 43, 17, 24]. It is curious to note that rainbow Ramsey problems have received great attention in the context of graph theory (see [12, 8, 2, 4, 36, 13, 6, 29, 20, 3, 28, 21] and references therein).

Ramsey's theorem states that if there are enough vertices, then at least one thing (e.g., red or blue triangle) is guaranteed to exist. The Ramsey number R(k,l) is defined as the smallest integer n such that in any two-coloring of the edges of Kn by red and blue, either there is a red Kk or a blue Kl.

We investigate a new generalization of the generalized ramsey number for graphs. Recall that the generalized ramsey number for graphs G1,G2,….., Gc is the minimum positive integer N such that any coloring of the edges of the complete graph K n with c colors must contain a subgraph isomorphic to G, in color i for some i. Bialostocki and Voxman defined RM (G) for a graph G to be the minimum N such that any edge-coloring of K n with any number of colors must contain a subgraph isomorphic to G in which either every edge is the same color (a monochromatic G) or every edge is a different color (a rainbow G). This number exists if and only if G is acyclic.

There are some other surveys of edge coloring that we should mention. The first is the dynamic survey [199] by Radziszowski which contains a wonderful list of known (monochromatic) Ramsey numbers. There is a brief survey of anti-Ramsey results in [207]. Also there is a survey by Kano and Li [140] which discusses some rainbow coloring. There is also a forthcoming survey by Fujita, Liu and Magnant [85] related to this survey but focusing more on large monochromatic structures. It should be noted that in [216], Voloshin demonstrates very interesting relationships between rainbow / monochromatic subgraphs and mixed hypergraph colorings. In fact, many of the notions of generalized Ramsey colorings are very closely related to upper and lower chromatic numbers of the derived mixed hypergraph.

All graphs considered in this paper are undirected and simple. Cm,Pm,Km and Sm stand for cycle, path, complete, and star graphs on m vertices, respectively. The graph Ki + Pn is obtained by adding an additional vertex to the path Pn and connecting this new vertex to each vertex of Pn. The number of edges in a graph G is denoted by ¿(G). Further, the minimum degree of a graph G is denoted by 6(G). An independent set of vertices of a graph G is a subset of the vertex set V(G) in

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which no two vertices are adjacent. The independence number of a graph G, a(G), is the size of the largest independent set. The neighborhood of the vertex u is the set of all vertices of G that are adjacent to u, denoted by N(u). N[u] denote to N(u) U{u}. For vertex-disjoint subgraphs H1 and H2 of G we let E(H1 ,H2) = {xy e E(G) : x e V(H1),y e V(H2)}. Let H be a subgraph of the graph G and U Q V(G), NH(U) is defined as (UueU N(u)) n V(H). Suppose that V1 Q V(G) and V1 is nonempty, the subgraph of G whose vertex set is V1 and whose edge set is the set of those edges of G that have both ends in V1 is called the subgraph of G induced by V1, denoted by (V1)G.

he cycle-complete graph Ramsey number r(Cm,Kn) is the smallest integer N such that for every graph G of order N, G contains Cm or a(G) > n. The graph (n - 1)Km-1 shows that r(Cm,Kn) > (m - 1)(n - 1) + 1. In one of the earliest contributions to graphical Ramsey theory, Bondy and Erdos [1] proved the following result: for all m > n2 - 2, r(Cm,Kn) = (m - 1)(n - 1) + 1. The above restriction was improved by Nikiforov [2] when he proved the equality for m > 4n + 2. Erdos et al. [3] gave the following conjecture.

Conjecture 1. r(Cm,K„) = (m - 1)(„ - 1) + 1, for all m > n > 3 except r (C3,K3) = 6.

The conjecture was confirmed by Faudree and Schepl [4] and Rosta [5] for „ = 3 in early work on Ramsey theory. Yang et al. [6] and Bollobas et al. [7] proved the conjecture for „ = 4 and „ = 5, respectively. The conjecture was proved by Schiermeyer [8] for „ = 6. Jaradat and Baniabedalruhman [9,10] proved the conjecture for „ = 7 and m = 7,8. Later on, Chena et. al. [11] proved the conjecture for „ = 7. Recently, Jaradat and Al-Zaleq [12] and Y. Zhang and K. Zhang [13], independently, proved the conjecture in the case „ = m = 8. In a related work, Radziszowski and Tse [14] showed that r(C4,K7) = 22 and r(C4,K8) = 26. In [15] Jayawardene and Rousseau proved that r(C5,K6) = 21. Also, Schiermeyer [16] proved that r (C5, K7) = 25. For more results regarding the Ramsey numbers, see the dynamic survey [17] by Radziszowski.

Rainbow arithmetic progressions in [n] and N

In 1916, Schur [39] proved that for every k, if n is sufficiently large, then every k-coloring of [n] := {1,...,n} contains a monochromatic solution of the equation x + y = z. More than seven decades later, Alekseev and Savchev [1] considered what Bill Sands calls an un-Schurproblem [18]. They proved that for every equinumerous 3-coloring of [3n] (i.e., a coloring in which different color classes have the same cardinality), the equation x+y = z has a solution with x, y and z belonging to different color classes. Such solutions will be called rainbow solutions. E. and G. Szekeres asked whether the condition of equal cardinalities for three color classes can be weakened [42]. Indeed,Sch¨onheim [38] proved that for every 3-coloring of [n],

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such that every color class has cardinality greater than n/4, the equation x + y = z has rainbow solutions. Moreover, he showed that n/4 is optimal.

Inspired by the problem above, the third author posed the following conjecture at the open problem session of the 2001 MIT Combinatorics Seminar [22], which was subsequently proved by the authors in [23].

Theorem 1 (Conjectured in [22], proved in [23].) For every equinumerous 3-coloring of [3n], there exists a rainbow AP(3), that is, a solution to the equation x + y = 2z in which x, y, and z are colored with three different colors.

In [22], Fox 1, Mahdian, and the authors proved the following infinite version of Theorem 1.

$$c(i) := \begin{cases} B & \text{if } i \equiv 1 \pmod 6 \\ G & \text{if } i \equiv 4 \pmod 6 \\ R & \text{otherwise} \end{cases}$$

Theorem 2 [22] Let c : N → {R, G, B} be a 3-coloring of the set of natural numbers with colors Red, Green, and Blue, satisfying the following density condition

lim sup n→∞ (min(Rc(n), Gc(n), Bc(n)) − n/6) = +∞

where Rc(n) is the number of integers less than or equal to n that are colored red, and Gc(n) and Bc(n) are defined similarly. Then c contains a rainbow AP(3).

Basically Theorem 2 states that every 3-coloring of the set of natural numbers with the upper density of each color greater than 1/6 admits a rainbow AP(3).

Based on the computer evidence and the intuitive belief that the finite version of Theorem 2 should be true as well, in [22], we posed as a conjecture the following stronger form of Theorem 1, which has been recently confirmed by Axenovich and Fon-Der-Flaass [5].

$$r(n) := \begin{cases} \lfloor (n+2)/6 \rfloor & \text{if } n \not\equiv 2 \pmod 6 \\ (n+4)/6 & \text{if } n \equiv 2 \pmod 6 \end{cases} \quad (1)$$

Theorem 3 (Conjectured in [22], proved in [5].) For every n ≥ 3, every partition of [n] into three color classes R, G, and B with min(|R|, |G|, |B|) > r(n), where

contains a rainbow AP(3). The following coloring of N:

contains no rainbow AP(3) and min(Rc(n), Gc(n), Bc(n)) = (n + 2)/6, hence showing that Theorem 2 is the best possible. Clearly, for n ≡ 2 (mod 6), this coloring shows that Theorem 3 is tight as well. As for the remaining case (when n = 6k + 2 for an integer k), we define a coloring c as follows:

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$$c(i) := \begin{cases} B & \text{if } i \leq 2k+1 \text{ and } i \text{ is odd} \\ G & \text{if } i \geq 4k+2 \text{ and } i \text{ is even} \\ R & \text{otherwise.} \end{cases}$$

Since every blue number is at most 2k + 1, and every green number is at least 4k + 2, a blue and a green number cannot be the first and the second, or the second and the third terms of an arithmetic progression with all terms in [n]. Also, since blue numbers are odd and green numbers are even, a blue and a green cannot be the first and the third terms of an arithmetic progression. Therefore, c does not contain any rainbow AP(3). It is not difficult to see that c contains no rainbow AP(3) and min(Rc(n), Gc(n), Bc(n)) = k +1=(n + 4)/6. The existing proofs of Theorems 1, 2, and 3 use a fact that every rainbow-free

coloring contains a dominant color, that is, a color x such that for every two consecutive numbers that are colored with different colors, one of them is colored with x. The rest is to show that under certain density conditions the dominant color is not excessively dominant, so a rainbow AP(3) exists.

One way to generalize Theorems 1 and 3 is to increase the number of colors and the length of a rainbow AP.

Axenovich and Fon-Der-Flaass came up with a construction for k ≥ 5, that no matter how large the smallest color class is, there is a k-coloring with no rainbow AP(k). Their construction is as follows [5].

Let n = 2mk, k ≥ 5. We subdivide [n] into k consecutive intervals of length 2m each, say A1,...,Ak and let t = k/2. Then,

$$c(i) = \begin{cases} j-1 & \text{if } i \in A_j \text{ and } j \neq t, j \neq t+2 \\ t-1 & \text{if } i \in A_t \cup A_{t+2} \text{ and } i \text{ is even,} \\ t+1 & \text{if } i \in A_t \cup A_{t+2} \text{ and } i \text{ is odd.} \end{cases}$$

It is easy to see that the above coloring does not contain any rainbow AP(k) and the size of each color class is n/k. For example, the coloringc in the case n = 60, k = 5, m = 6, t = 2, is as follows.

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However, the case k = 4 is still unresolved.

In this paper we confirm the Erdos, Faudree, Rousseau, and Schelp conjecture in the case C9 and K8. In fact, we prove that r(C9,K8) = 57. It is known, by taking G = („ - 1)Km_i, that r(Cm,K„) > (m - 1)(„ - 1) + 1. In this section we prove that this bound is exact in the case m = 9 and „ = 8. Our proof depends on a sequence of 8 lemmas.

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Lemma 2.1. Let G be a graph of order > 57 that contains neither C9 nor an 8-elemant independent set. Then 6(G) > 8.

Proof. Suppose that G contains a vertex of degree less than 8, say u. Then |V(G - N[u])| > 49. Since r(C9,K7) = 49, as a result G - N[u] has independent set consists of 7 vertices. This set with the vertex u is an 8-elemant independent set of vertices of G. That is a contradiction. □

Throughout all Lemmas 2.2 to 2.8, we let G be a graph with minimum degree 6(G) > 8 that contains neither C9 nor an 8-elemant independent set.

Lemma 2.2. If G contains K8, then |V (G)|> 72.

Proof. Let U = {u1 ,u2,u3,u4,u5,u6,u7,u8} be the vertex set of K8, Let R = G - U and Ui = N (ui) n V (R) for each 1 < i < 8. Since 6(G) > 8, Ui / 0 for all 1 < i < 8. Since there is a path of order 8 joining any two vertices of U, as a result Ui n Uj = 0 for all 1 < i < j < 8 (otherwise, if w e Ui n Uj for some 1 < i < j < 8, then the concatenation of the uiuj-path of order 8 with uiwuj, is a cycle of order 9, a contradiction). Similarly, since there is a path of order 7 joining any two vertices of U, as a result for all 1 < i < j < 8 and for all x e Ui and y e Uj

we have that xy / E(G) (otherwise, if there are 1 < i < j < 8 such that x e Ui, y e Uj and xy e E(G), then the concatenation of the uiuj-path of order 7 with uixyuj, is a cycle of order 9, a contradiction). Also, since there is a path of order 6 joining any two vertices of U, as a result, NR(Ui) n NR(Uj) = 0, 1 < i < j < 8 (otherwise, if there are 1 < i < j < 8 such that w e NR(Ui) n NR(Uj), then the concatenation of the uiuj-path of order 6 with uixwyuj, is a cycle of order 9 where x e Ui, y e Uj and xw,wy e E(G), a contradiction). Therefore |Ui U NR(Ui) U {ui}| > 6(G) + 1. Thus, |V(G)| > 8(6(G) + 1) > (8) (9) = 72.

If G contains K8 - S6, then G contains K8.

Proof. Let U = {ui, u2, u3, u4, u5, u6, u7, u8} be the vertex set of K8 - S6 where the induced subgraph of {u1f u2, u3, u4, u5, u6, u7} is isomorphic to K7. Without loss of generality we may assume that u1u8,u2u8 e E(G). Let R = G - U and Ui = N(ui) n V(R) for each 1 < i < 8. Then, as in Lemma 2.2, we have the following:

(1) Ui n Uj = 0 for all 1 < i< j < 8 except possibly for i = 1 and j = 2.

(2) E(Ui, Uj) = 0 for all 1 < i<j < 8.

(3) NR(Ui) n NR(Uj) = 0 for all 1 < i<j < 8.

(4) E(Nr(U{),Nr (Uj)) = 0 for all 1 < i<j < 8.

Since a(G) < 7, as a result at least five of the induced subgraphs (Ui U NR(Ui))G, 3 < i < 8 are complete. Since 6(G) > 8, it implies that these complete graphs contain K8. Hence, G contains K8.

CONCLUSION

. The theme of this paper was to efficiently force a rainbow copy of a specific graph H in every proper coloring of a constructed graph G. We have measured efficiency by the number of edges required in G and have considered this both in the online and offline setting. Let us mention another concept of efficiency that might be interesting to study: For given graph H what is the least maximum degree of a graph G with G → H, i.e., where every proper edge-coloring of G contains a rainbow copy of H? Clearly, the maximum degree of G must be at least |E(H)|−1 since otherwise G can be properly colored with less than |E(H)| colors and hence does not contain a rainbow copy of H. There is also an online variant of this question, which is analogous to the online antiRamsey numbers we defined here. One can show that Builder can force a rainbow matching on k edges even if the graph she presents has maximum degree at most d(k + 1)/2e and that no rainbow copy of any k-edge graph H can be forced by Builder if she is restricted to presenting a graph of maximum degree strictly less than d(k + 1)/2e.

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