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# Controller Assisted Design: An Interactive Simulation of a Multirate Control Strategy for Dealing with Slow Sensors

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## Abstract:

*The sensor technology used to measure the controllable variable cannot often keep a fixed sample time in many control applications. The assumption of a consistent and random sampling strategy is problematic here. In addition, a multirate controller is often the answer if the control action update can be quicker than the output measurement frequency to realize the required closed loop behaviour. When designing a multirate (MR) system, it's important to keep in mind a few well-known caveats. Input-output sampling period multiplicity, controller design, ripple presence, and other factors must all be taken into account. Having access to a computer-assisted design tool is a great approach to save time and get professional results. It would seem that the best way to handle MR is using a tool for interactive simulation. This work introduces a simulation program of this kind. When the settings of the multirate sampling pattern are adjusted, the resulting change in performance may be easily understood. Sysquake, a language similar to MATLAB, was used to create the tool because of its fast execution and robust graphical capabilities. It may be sent to the recipient in executable form. In addition to providing a comprehensive overview of MR therapy, this article will also discuss the design of four distinct MR controllers that have a modular structure that allows them to be changed to various treatment plans. When delays in time occur, these MR systems additionally make use of the Smith's predictor, which is discussed, defended, and implemented. Finally, some intriguing findings made with this dynamic resource are provided.*

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## Introduction

In a multirate sampling (MR) system, two or more variables are sampled or updated at various frequencies, making it a hybrid system made up of continuous time elements (typically the plant) and some discrete time components (typically the controllers or the filters). Discrete events may also be delayed relative to one another or not occur at regular intervals. In addition, the assumption of a regular pattern of sampled signals is made in a great many computer control applications. The procedure may be made simpler by making the non-very

restrictive assumption that the sample pattern is periodic. In other words, there is a global period  $T_0$  with cyclic recurrence, but the process variables are sampled and/or updated at distinct and/or irregular intervals. Despite the potential for a lag between sampling and variable updates, it is still reasonable to assume global periodicity. This study will not address the far more difficult issue of asynchronous sampling/updating, in which the discrete operations occur at random.

Although it is generally assumed that the variables in a digital control system will be sampled and updated in a perfectly uniform fashion, it is important to note that in real-world applications, the synchronicity of the set of discrete actions is often not perfect and can be adjusted to achieve better results. Therefore, MR is a pressing concern from both the academic and the professional communities. Users should be able to quickly grasp the implications of MR's presence in a variety of contexts. For a real-time process control request, the

The theoretical study of controlled system performances, however, is much more computationally complex in such situations. Much of an engineer's workday may be taken up by the phases of modelling, analysis, and design. Time and frequency approaches and tools are often used to investigate and research the many properties of the dynamic behaviour of an MR. These will highlight the relationship between the various regulated plant performances and offer a comprehensive picture of the system's behaviour.

### Conceptual Groundwork

The preceding section laid forth the goals and parameters of this study. In this part, we will present the issue at large, the necessary terminology, and the fundamental operations between signals and processes. After detailing the kind of challenges encountered by practitioners who give this issue serious thought, the fundamental signal-frequency-operations and their attributes are introduced. Some elementary transformations between polynomials and the available relations between fast-skipped and slow or slow-expanded and fast signals are also discussed in this section, as are the notations used in process transfer functions within the MR context. Discrete lifting is then fitted to our algebra, which is how it is often presented in the literature. A survey segment that follows the design process described in segment 3. To begin, it is important to highlight that the systems discussed in this article are referred to as MultiMate Systems, which are defined as systems including sampled or discrete signals referenced to two or more distinct frequencies. The many problems associated with such systems may be better grasped with the aid of a preliminary framework, as shown in Figure 1.

amount of time needed for chemical tests or samples gathered using artificial vision with post-processing needs might be substantial. Distillation columns, unmanned aerial vehicles, network-based control systems, and so on all face the same issue when sensors are located in different physical locations than the controller algorithm device. The goal of the controller is to provide results that are comparable to those produced by the more rapid single rate controlle



Figure 1. An initial MR System.

One option in order to describe the different signals and systems in these environments is to use notation with superscripts. The signal (or system, when it is the case) denotes either the Z-transform of the sequence  $y(kT)$  derived from the sampling with period  $T$  of the continuous signal  $y(t)$ :

$$Y^T \triangleq Z \{y(k)\} = \sum_{k=0}^{\infty} y(kT)z^{-k}$$

or the sampling rate transformation of a discrete signal  $Y$  (as will be explained below). With respect to Figure 1:

$$Y^{NT} = [G(s)U^{MT}]^{NT} = G^{NT} [U^{MT}]^{NT}$$

where represents the continuous process discretization (usually ZOH-discretization) at period  $NT$ :

$$G^{NT} = Z \left[ \frac{1 - e^{-NTs}}{s} G(s) \right]$$

This single example enables one to understand that the sampling period transformation between discrete signals or the sampling operations involving blocks of different nature is quite common in MR systems. One way to achieve a proper handling of this kind of systems requires management of the great common divisor (gcd) and the least common multiple (lcm) of every sampling period occurring in the studied MR scheme. With

these magnitudes, every sampling period is going to be repeated an integer multiple of times in one lcm,  $T_0$ . There also will be a base period (gcd),  $T_B$  such that  $T_B = T_0/P$ , being  $P$  an integer greater than one. With respect to Figure 1, if it is called  $T_0$  the lcm period and

$$B = \text{lcm}(M, N), T_B = T_0/B.$$

After introducing the broader MR issue, we shall proceed with the assumption that the output sampling period is strictly larger than the input sample period (the multirate input control case, or MRIC). If an MR system is being investigated, it is sufficient to assume two sample periods, input  $T$  and output  $NT$ , where  $N$  is a positive integer higher than 1.

### Definition of the Issue

This work addresses the issue that arises when a control algorithm implementation of a  $T$  single rate control in a computer control application fails. The limited range of the sensor's frequency is the primary contributor. The optimal sample duration is impractical in many applications, including the chemical industry, artificial vision, network-based, and dispersed installations, since the sensors need a specific amount of time (for computing requirements or owing to sensor geographic location). In such conditions, using an MR control may be helpful. Solving this issue involves measuring the controlled variable  $N$  times more slowly to get the same  $T$ -behaviour. The need for a non-traditional controller that takes a slow input and produces a quick output is intuitive. Figure 4 depicts the fundamental system, which includes the introduction of a dual-rate control. The plant is modelled as an LTI continuous system (CT) of order  $n$ , with a transfer function of  $G_p(s)$ .

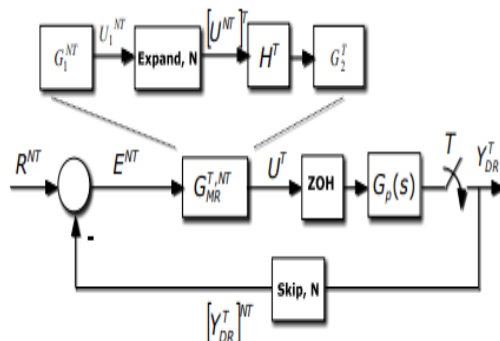


Figure 2. Dual rate controller structure.

Based in this MRIC structure, that is slow measurement and fast control updating, in [8] a new non-conventional structure composed by slow

and fast parts for the dual rate controller was exposed; an expand operation is required to assure the composition of two different frequency elements. The rest of the closed loop performs as follows: the controller output is updated at a period  $T$  through the fast hold device,  $H^T$ , and the system output,  $y(t)$ , is measured at period  $NT$ , being represented by a fast sampler followed by an  $N$ -sampler skip operation, and compared to the reference sampled at the slow rate,  $R^{NT}$ . The dual rate controller,  $G_{MR}^{T, NT}$ , processes the error at slow rate,  $E^{NT}$ , and generates  $N$  fast control actions,  $U^T$ , inside the meta period  $NT$ . It must be noted that just the case where in all blocks input and output sampling periods are integer is considered; that is, the more complex case when rational ratio appears is out of our scope.

### Model Based Controller In this case

the design procedure is based on a continuous closed loop including a controller (usually, a PID-type controller). From this controller a closed loop transfer function  $M(s)$  is obtained. Then, ZOH-discretization assuming periods  $T$  and  $NT$  must be computed in order to obtain  $H^T$  and  $G_2^T$ , and respectively. So, applying (32)–(34) the design step is completed. If the process is non-minimum phase, the cancellation of unstable pole-zero pairs must be avoided. Thus, the fast part of the controller could be alternatively computed by (35):

### Description of the Application

To better understand the multi-rate control aspects previously presented, an interactive simulation application has been developed. An executable version can be downloaded online from [22]. The main goal of this section is to detail the main features of this application, which has been programmed using an interactive simulation tool named Seaquakes [6]. Seaquakes uses a MATLAB-like programming language, which is provided with some specific commands in order to perform a high level of interactivity in the developed application. This fact enables students to more easily learn about advanced, complex control concepts and techniques, and researchers to exploit and improve their achievements [23–25]. Being aware interactivity is difficult to be shown in a writing text, next the different sections of the interactive application will be described. The application user interface presents a main window which is split into two parts (see in Figure 8): parameter section (upper left-hand part) and graphic section (lower left-hand part, and right-hand part). In addition, at the top side, the interface shows a menu bar and a toolbar, and at the bottom side, a status bar. These bars are provided by Seaquakes and their working

mode is standard, except for the Settings menu, which can be intentionally defined by the programmer.

### Settings menu:

by means of the option Controller of this menu, one of the four multi-rate controllers presented in the previous sections (PID controller, Cancellation controller, RST controller, or Model-based controller) can be chosen. Moreover, increments/decrements for every slider included in the interface can be defined by means of the option Slider Config of this menu, which provides two variation modes for the sliders: by percentage or by fixed value.

### Parameter section:

this section (located in the upper left-hand part; see Figure 8) enables to introduce the input data required in each case. The section is split into two parts in order to consider input data of different nature: process parameters (right-hand part) and controller parameters (left-hand part)

### Examples

In this section an example for each controller will be presented. Output responses for the single-rate and multi-rate cases will be compared and analysed, focusing on multi-rate control benefits.

### Model-Based Controller

Let us consider the example shown in Figure 9, where the process is defined by:

$$G_p(s) = \frac{1.5}{(s + 0.5)(s + 1.5)} e^{-0.5s}$$

An acceptable continuous-time PID controller is given by

$$K_p = 8, T_D = 0.2, T_I = 3.2$$

$$u(t) = K_p \left[ e(t) + T_D \dot{e}(t) + \frac{1}{T_I} \int_0^t e(\tau) d\tau \right]$$

which yields:

$$G_r(s) = 8 \frac{(s + 5)(s + 0.31)}{s}$$

From (49)–(50) a continuous-time closed-loop transfer function  $M(s)$  can be calculated. Then, if

the sampling time is  $T = 0.1$  s, and the multiplicity is  $N = 4$ , the dual-rate controller is designed from (32)–(34) (note that the single-rate controllers can be designed using (32) and (33) with the specific rate for each controller). Responses for each case are illustrated in Figure 9, where stable outputs are always observed due to the use of the Predictor options to face the process time delay ( $D = 0.5$ ). In this example, the dual-rate controller is able to achieve a response which is quite similar to that obtained by the fast-rate controller when following a step reference. However, when applying a load disturbance, the dual-rate output worsens with respect to the fast-rate one, since the dual-rate controller is designed for step references and not for impulse inputs like the load disturbance is. Regarding the slow-rate output, it is clearly worse for the both cases (when following a step reference and when applying a load disturbance), since it shows around 10% higher overshoot and 2 times longer settling time with respect to the dual-rate output. In any case, every output shows oscillations (ripple) due to cancelling the process dynamics. Deactivating the checkbox Oscillations, the controllers are designed following (35), and then the ripple effect totally disappears (see Figure 12). Moreover, in this case, whereas the slow-rate output is on the verge of instability, the dual-rate output presents around 5% less overshoot than the fast-rate output when following a step reference.

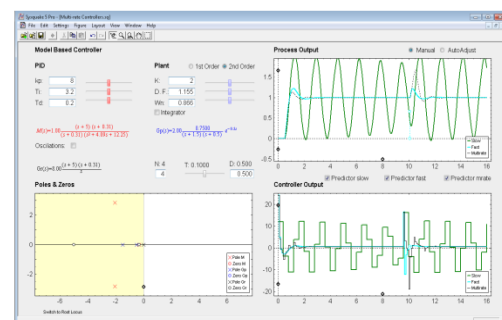


Figure 3. Application user interface for the Model-based (PID) controller with no oscillations

### Conclusions

One solution for the problem of control schemes with slow sensors is to assume a MR system, considering a restricted slow measurement sampling but also a faster control updating. the use of an interactive simulation tool in order to study multirate systems is a feasible option to make proper decisions about the correct control design. Different

non-conventional controller structures have been provided. From application some unexpected results are achieved and explained. Even for an expert in this field the tool appears absolutely essential and time saving. For a beginner student or researcher, it is entirely necessary if the study of this kind of systems is needed/desired.

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