ISSN: 2454-9940

INTERNATIONAL JOURNAL OF APPLIED SCIENCE ENGINEERING AND MANAGEMENT

E-Mail :
editor.ijasem@gmail.com
editor@ijasem.org

www.ijasem.org

INTERNATIONAL JOURNAL OF APPLIED SCIENCE ENGINEERING AND MANAGEMENT

ISSN2454-9940
www.ijsem.org
Vol 9, Issuse. 3 July 2018

Overview of Groups

Mrs V. Sandhya Rani M.Sc. (Mathematics); B.Ed.*1, Mr. M. P. Srinivas Rao, M.Sc. (Mathematics) *2 Mrs. Mayuri Odela, M.Sc. (Mathematics); B.Ed *3

ABSTRACT

Group Theory, a branch of abstract algebra, provides a powerful framework for understanding symmetry and mathematical structures. This article serves as a comprehensive introduction to the fundamental concepts of Group Theory and explores its diverse applications across various disciplines in diverse fields.

KEYWORDS: Abstract algebra, group theory, mathematical structure

Sets

Please read Introduction to Sets before continuing so that you are familiar withconcepts such as these:

Outfit: { blouse, coat, slacks,...}

a collection of even numbers {..., 0, 2, 4, -2, -4,...}

Positive three-fold multiples smaller than ten: {3, 6, 9}

Operations

It's nice to work with the elements of sets now that we have them. To be moreprecise, we want to integrate them somehow. An operation is used for this purpose.

An operation takes elements from a set and creates a new element bycombining them in some way. Or, to put it more plainly:

*1. Lecturer, Dept. of Mathematics, Siva Sivani Degree College, Kompally, Sec'Bad- 100
*2. HOD, Dept. of Mathematics, Siva Sivani Degree College, Kompally, Sec'Bad- 100

*3. Lecturer, Dept. of Mathematics, Siva Sivani Degree College, Kompally, Sec'Bad- 100

ISSN2454-9940
www.ijsem.org
Vol 9, Issuse. 3 July 2018

An Operation combines members of a Set.

"Insert" could be the operation for the clothing above. The socks can be putinto the shoes.

You could even put the socks inside the shoes.

And I can give you artists out there an example with painting. Assume that wehave the color palette {red, green, blue}.

It is now necessary to define an operation, and mixing is the most logicalchoice.

Red combined with green, for instance, yields yellow, and red combined withblue, purple.

However, it is tedious and long to say, "Red mixed with blue makes purple." If Ihave to write a lot, I will try to make it shorter. Thus, I'm going to use the symbols + for "mixed with" and = for "makes."

Therefore, "red + blue = purple" replaces "red mixed with blue makes purple."

Binary Operations

We have been a little too general thus far. Thus, we will now get a little moredetailed. Similar to an operation, but requiring two elements, is a binary operation. And merges them into one, neither more nor less.

You may not be aware of it, but you already know a few binary operators.Familiarize yourself with them:

5 + 3 = 8

4 x 3 = 12

4 – 4 = 0

These all take two numbers and generate one number by combining them invarious ways.

See the final example, where 4 - 4 = 0. Even if the two elements are the same,it still requires two.

(Additionally, since division also yields a remainder, it is not included.)

It appears that there are three operations above. In a moment, you willdiscover that

ISSN2454-9940
www.ijsem.org
Vol 9, Issuse. 3 July 2018

there are actually just two!

Well Defined

Operators have one requirement: they have to be precisely defined. However,turn that around. They must be precisely defined.

Consider incorporating the terms "defined well" into the English language. Should a word has a clear definition, so you can understand what I mean whenI use it.

Since you are aware of exactly what I mean when I use the word "angry," it isdefined fairly well. But is a piece of fruit or a date on a calendar what I mean when I say the word "date"?

Let's put this to use now! If I were to provide you with two numbers and a clear operation, you ought to be able to provide me with the precise outcome.
For instance, $5 + 3$ has only one possible response. This is as a result of theoperator's clear definition.

However, there are some ill-defined objects that appear to be operators.Square roots are one example. There are two solutions when we write x^2 = 25, or more accurately, $x = \pm \sqrt{(25)}$.
I could simply reply, "Nope, the answer is −5," if you tell me the answer is 5.You're not correct. Since $(-5) \times (-5) = 25$ and $5 \times 5 = 25$.

There is only one possible response when using well-defined operators.

Finally, a note on operations: * is frequently used to indicate an operation. Though we can certainly use it for that, multiplication is not what we mean. However, typically, we only refer to "some operation." We make it clear whenwe mean multiplication.

Overview of Groups

You are aware of the fundamental components that comprise groups now thatwe have discussed sets and operators. To put it plainly:

A group is a Set combined with an operation

Take the set of integers with addition as an example.

ISSN2454-9940
www.ijsem.org
Vol 9, Issuse. 3 July 2018

However, it is a little more intricate than that. If all we know is that there is a set and an operator, then we cannot say much. What more could we say? Werequire additional details regarding the operator and the set. Groups are subject to restrictions for this reason. They have more properties, in other words.

Formal Definition of a Group

A group is a set G plus an operation * that results in:

1. There is an identity within the group.

2. There are inverses in the group.

3. It's an Associative operation.

4. The group is closed under the operation.Let's examine each of those separately:

1. There is an identity within the group. Should we apply the function to any component and the identity, that component will be returned to us.

The identity for addition and integers is "0". Since 0+5 = 5 and 5+0 = 5,

Put another way, when combined with other elements, it doesn't changethem.

Every group has a single element of identity.

The identity element's symbol is e, or occasionally 0. But instead of viewing0 as a number, you must begin to see it as a symbol. Similar to how e is theidentity symbol, 0 is also that. It's how it's defined. In fact, since 0 is so much more natural than e, mathematicians frequently choose to use it instead of e.

Formal Statement :

In the set G, there is an e such that for every element a in G, a * e = a ande * a = a.

2. There are inverses in the group. If there is a group component, there's an additional group element such that the identity, e, is obtained whenthe operator is applied to both of them.

INTERNATIONAL JOURNAL OF APPLIED SCIENCE ENGINEERING AND MANAGEMENT

ISSN2454-9940
www.ijsem.org
Vol 9, Issuse. 3 July 2018

The inverse of five is five in terms of addition and integers. (since -5 + 5 = 0)

Similarly, the inverses of negative integers are positive. Five plus five equalszero. Hence, 5 is the inverse of -5. Actually, since a is equal to b in inverse form, b must be the opposite of a.

Reverses are distinct. No other number x that can be named so that 5 + x = 0aside from -5. Note that each element in the group has a unique inverse, even though thereis only one identity for every single element in the group.

We represent inverses using the notation a^-1. Thus, a^-1 = b in the exampleabove. Likewise, 5^-1 = -5 applies when discussing addition with integers.

Formal Statement:

Let a * b = e and b * a = e for every a in G. There exists b in G.

3.Associative : Associative laws are something you should have studied in elementary algebra. It simply indicates that there is no significance to thesequence in which we perform specific tasks.

a * (b * c) = a * (b * c)

Observe that we proceeded a, b, and c. Just the parentheses were altered.We'll discuss this again later.

Formal Statement:

a * (b * c) = (a * b) * c for all a, b, and c in G.

4.Closed under the Operation. Picture yourself enclosed in a large box. It isimpossible to go outside when you are inside. Similarly, after you have twogroup members, whatever the elements may be, utilizing the operation onthem won't cause you to leave the group.

It follows that if there are two elements in the group, a and b, then a * b mustalso be in the assembly. This is our definition of closed. It is referred to as closed because, from within the group, we are unable to leave it.

ISSN2454-9940
www.ijsem.org
Vol 9, Issuse. 3 July 2018

The same holds true for addition and integers as it did for the previously mentioned properties. Since x + y = z implies that both x and y are integers, zmust also be an integer.

Formal Statement:

a * b is in G for all elements a, b in G.

Thus, you have a group if you have a set and an operation and you are ableto meet each of those requirements**.**

Just two procedures

I demonstrated the four distinct operators we employ with the numbers we are accustomed to way back near the top:

− + × /

However, there are only just two procedures!

Because b is short, we use "a minus b" when subtracting numbers. Nevertheless, "a plus the additive inverse of b" is indeed what we intend.

In actuality, the minus sign just indicates to add the additive inverse. We merely say "minus" since it would be absurd to keep repeating that.

What is division, in your opinion? Likewise, it simply means "multiply by the multiplicative inverse."

Thus, the only operations available are addition and multiplication!

Give examples!

Oh my! Bewildered? Most likely, you are. This is where they become useful.

Example: {0} with addition

This is a strange example, though. However, let's give the three steps a try. Now let's locate the identity component. That shouldn't be too difficult, though. Our goal is to obtain 0 if we add 0 to anything else in the group. We have discovered the identity as 0 is the only other element in the group and 0 + 0 = 0.

Finding inverses is now necessary. Once more, there is just one component. What then is 0's inverse? Our goal is 0 + 0-1 = 0. 0-1 equals 0, since 0 + 0 = 0. Additionally, since 0 is a member of the group, 0-1 is as well. After attempting each component, all

Multiplication and {-1, 1} as an example

Rewind to the initial four stages. Is there an identity first? This should be simple as there are

ISSN2454-9940
www.ijsem.org
Vol 9, Issuse. 3 July 2018

just three options. Alternatively:

The identification can be either -1, +1, or nonexistent.

Both 1*-1 and -1*1 equal -1. Thus, 1 appears to be the identity. ought to have anticipated that.

Finding inverses is now necessary. Find an a-1 such that a * a^{-1} = 1 (or, more accurately, e) if there is an in the group. Thus, let's begin with 1.

We know that if a = 1, then a-1 = 1 as well because 1 * 1 = 1. -1 * -1 = 1 now. Therefore, a-1 = -1 if a = -1.

Example: Addition and integers

Think about the whole numbers. When it comes to addition, can you name the identity element of integers? Finding a + e = e + a = an is our goal. Alright, you already know. 0 represents the identity. This is thus because, for any integer a, a + 0 = 0 + a = a.

Let's continue working with integers and assume we have a number a. Is its inverse known?

Is there an a-1 such that a + a-1 = a-1 + a = e, in other words? For instance, 5 + 5 + 1 = 0? 5-1: What is it? The solution is -5. To the integers, a + -a = e.

Will the sum of two integers be an integer if I add them together? Indeed. It is hence closed.

Why Do Groups Exist?

Why then do we find these groups important? Answering that question is difficult, though. Not that there isn't a decent one; rather, because group applications are so sophisticated.

They are used, for instance, on your credit cards to verify that the scanned numbers are accurate.

Space probes employ them so that misinterpreted data can be corrected. They are even employed in determining whether polynomials have solvable solutions.

Here's a valid explanation:

Unique Categories of Groups: Abelian

Let me clarify that the name Abelian is pronounced abelian before I continue.

Not Abelian. When I initially started reading about organizations, I made that error, and I still haven't broken the habit.

It bears the name Niels Henrik Abel in honor of the mathematician.

Doesn't e * a = a follow if a * e = a?

In a similar vein, doesn't b * a = e follow if a * b = e?

In actuality, though, it does. However, we must use caution because, generally speaking, a * b does not equal b * a. In contrast, if it is true that for every a and b in the group, a * b = b * a,

ISSN2454-9940
www.ijsem.org
Vol 9, Issuse. 3 July 2018

then it is called abelian group.

References:

1"Abstract Algebra" by David S.Dummit and Richard M.Foote

2. A course in the theory of groups by DEREK J.S.Robinson