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Pascal matrices are helpful for calculations that require precision and conditioning.

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ABSTRACT

We show a theorem about Pascal matrices that are in an unsuitable state. The use of bidiagonal factorizations of Pascal matrices, however, is shown as a means to achieve relatively high precision in numerical calculations.

INTRODUCTION

Pascal matrices have been around for a while (see to [1-3]) and have found use in a wide variety of disciplines, including probability, combinatory, numerical analysis, electrical engineering, and image and signal processing (refer to [2,4] and [5,6]). Solving linear systems using rapid methods has been the subject of numerous recent articles. The use of Pascal matrices (see to [6]) and quick eigenvalue methods (cf. [7]). The ill-conditioning of Vandermonde matrices is well-known to grow with matrix size. From the Matrix. In Section 3, we use the Skeel condition number to verify a result about the ill-conditioning of Pascal matrices.

Proving that, of a given order, they are always less well conditioned than an equivalent Vandermonde matrix. But even with this outcome, we prove that accurate methods can be obtained for calculating eigenvalues and inverses of Pascal matrices, and for solving linear equations using Pascal matrices as coefficients. HRA denotes regardless of the condition's magnitude, the relative errors of the calculations are on the order of machine accuracy. Number When the calculations don't need subtractions (apart from the original data), as is well known (cf. [8]), the Legal protections for HRA are available. Two resources are used in the development of HRA algorithms for Pascal matrices. For one thing, a specifically, the bidiagonal factorization of Pascal matrices is discussed in Chapter 2.

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Factorization of Pascal metric

The symmetric matrix is an n-by-n Pascal matrix.

$$P = (p_{ij})_{1 \le i,j \le n}; \quad p_{ij} := \binom{i+j-2}{j-1}. \tag{1}$$

Pascal matrices of rank n are lower triangular matrices.

$$P_L = (q_{ij})_{1 \le i,j \le n}; \quad q_{ij} := \binom{i-1}{j-1}, \tag{2}$$

where $q_{ij} := 0$ if $j > i$.

It is widely known (see to [10]) that the factor of the Cholesky factorization of the Pascal matrix P is the lower triangular Pascal matrix PL:

$$P = P_L P_L^T. \tag{3}$$

A major component of the precise methods employed in this article is the following well-known result (cf. Lemma 1(ii) of [9]), which offers a bidiagonal decomposition of a lower triangular Pascal matrix. We will provide a proof because it is important to do so.

Lemma 1. Given (2), we get a lower triangular Pascal matrix of order n PL that satisfies

$$P_L = \begin{pmatrix} 1 & & & & & \\ 0 & 1 & & & & \\ & \ddots & \ddots & & & \\ & & 0 & 1 & & \\ & & & 1 & 1 & \end{pmatrix} \begin{pmatrix} 1 & & & & & \\ 0 & 1 & & & & \\ & \ddots & \ddots & & & \\ & & 1 & 1 & & \\ & & & 1 & 1 & \end{pmatrix} \cdots \begin{pmatrix} 1 & & & & & \\ 1 & 1 & & & & \\ & \ddots & \ddots & & & \\ & & 1 & 1 & & \\ & & & 1 & 1 & \end{pmatrix}. \tag{4}$$

Proof. Let F_i $(1 \le i \le n-1)$ be the $n \times n$ matrix

$$F_i = \begin{pmatrix} 1 & & & & & & \\ 0 & 1 & & & & & \\ & \ddots & \ddots & & & & \\ & & 0 & 1 & & & \\ & & & 1 & 1 & & \\ & & & & \ddots & \ddots & \\ & & & & & 1 & 1 \end{pmatrix} \begin{matrix} \\ \\ \\ \leftarrow (n-i+1)\text{th row} \\ \\ \\ \\ \end{matrix}$$

and let $P_L^{(k)} = (q_{ij}^{(k)})_{1 \le i,j \le n}$ be the matrix $P_L^{(k)} = F_1 F_2 \cdots F_k$.
Let us prove by induction on k that

$$q_{ij}^{(k)} = q_{i-1,j-1}^{(k)} + q_{i-1,j}^{(k)}, \quad 1 \le i \le n,\ n-k \le j \le n, \tag{5}$$

defining $q_{i0}^{(k)} = q_{0j}^{(k)} = 0$, with $1 \le i, j \le n$ and $q_{00}^{(k)} = 1$.

Given that P(k) L is a lower triangular matrix with a unit diagonal, it follows that (5) holds true k for I j, in particular for the final column of P(k) L. Simply demonstrating that (5) is true for I > j for j = no, nk+1,... n1 is all that has to be done. If k = 1, then it must be true. Let's start off by assuming that (5) is true for k 1, and then show it for k. Recall that P (k) In order to achieve L, one needs just consult P(k−1) L by multiplying each column by nk, then nk+1, etc., until n1 is reached.

If $n - k \le j \le n - 1$, then

$$q_{ij}^{(k)} = q_{ij}^{(k-1)} + q_{i,j+1}^{(k-1)}, \quad \forall i \tag{6}$$

And if one accepts the induction hypothesis, then for I > n, one obtains

$$q_{ij}^{(k)} = q_{i+1,j+1}^{(k-1)}, \quad n-k \le j \le n-1. \tag{7}$$

Using the assumptions that n is less than or equal to k, j, and n is less than or equal to n and that I is less than or equal to n, we may get (5). For k = n 1, we will use the result from (5) to show that

$$q_{ij}^{(n-1)} = (F_1 F_2 \cdots F_{n-1})_{ij} = \binom{i-1}{j-1}, \quad \text{if } i \ge j,$$

To clarify, qij of (2). Proof is accomplished using induction on the rows I of P (n1) L for I = 1, n. For the first set of cells, the inductive assertion is undeniably correct. Let's take it as given that it holds for 1,.., I 1, and then demonstrate that it holds for i.

If $1 < j < i$, by (5) we have $q_{ij}^{(n-1)} = \binom{i-2}{j-2} + \binom{i-2}{j-1} = \binom{i-1}{j-1}$.

If $j = 1$, then $q_{i1}^{(n-1)} = q_{i-1,1}^{(n-1)} = \binom{i-2}{0} = 1 = \binom{i-1}{0}$. Finally, if $i = j$, $q_{ii}^{(n-1)} = 1$, and the proof is finished. □

When used in conjunction, factorizations (3) and (4) produce a bidiagonal decomposition of a matrix P, which we'll refer to as BD (P). Since it is intuitive that all elements of a Pascal matrix are positive, this factorization may also be used as evidence that the matrix is nonnegative in its whole. Always positive and it is generally known (see Theorem 3.1 of [11]) that a combination of completely positive matrices is likewise completely positive.

Conditioning and accurate algorithms for Pascal matrices

To begin, let's review what what a Vandermonde matrix is. The matrix is a Vandermonde matrix of order n.

$$V = (v_{ij})_{1\leqslant i,j\leqslant n}; \quad v_{ij} := (j^{i-1}). \tag{8}$$

Vander monde matrices are notorious for their poor state.

We refer to the matrix whose I j)-entry is |aij| if A = (aij)16i,j6n. Let us look at the conventional and Skeel condition numbers of a nonsingular matrix A, represented by (A) and Cond(A), respectively, and defined by

$$\kappa_\infty(A) := \|A\|_\infty \|A^{-1}\|_\infty, \qquad \mathrm{Cond}(A) := \|\,|A^{-1}|\,|A|\,\|_\infty.$$

Two characteristics are worth remembering:

- $\mathrm{Cond}(A) \leqslant \kappa_\infty(A)$ In addition to its little size,
- in contrast to $\kappa_\infty(A)$, $\mathrm{Cond}(A)$ Is row-invariant; if D is a nonsingular diagonal matrix,

$$\mathrm{Cond}(DA) = \mathrm{Cond}(A).$$

Some of the reasons why the Skeel condition number Cond(A) is preferable to the classical condition number (A) are given by these features (cf. also Section 7.2 of [12]). Let's show that, regardless of order, Pascal matrices always exhibit poorer conditioning than Vander monde matrices.

Let P and V denote the Pascal and Vander monde matrices of order n from equations (1) and (8), respectively, and prove the following theorem. Then

$$\mathrm{Cond}(V) \leqslant \mathrm{Cond}(P). \tag{9}$$

Proof. Let T be the satisfiable matrix.

$$P = TV. \tag{10}$$

According to Theorem 3 of [13], if T > 0 and T te = e, where e = (1, 1, 1) t, then T is a stochastic matrix (cf. also [14]). We may create the following diagonal matrices with positive diagonal elements given that P and V are nonnegative matrices with nonzero row sums:

$$D_1 := \mathrm{diag}\left(\sum_{j=1}^{n} p_{1j}, \sum_{j=1}^{n} p_{2j}, \ldots, \sum_{j=1}^{n} p_{nj}\right), \quad D_2 := \mathrm{diag}\left(\sum_{j=1}^{n} v_{1j}, \sum_{j=1}^{n} v_{2j}, \ldots, \sum_{j=1}^{n} v_{nj}\right).$$

Next, notice that matrices

$$\bar{P} := D_1^{-1}P, \quad \bar{V} := D_2^{-1}V \tag{11}$$

Comprise random matrices. To put it another way, by using (10) and (11),

$$\bar{P} = (D_1^{-1}TD_2)\bar{V}. \tag{12}$$

Let us define

$$H := D_1^{-1}TD_2. \tag{13}$$

Without a doubt, H is a positive number. As (12) and (13) show, because P and V are stochastic matrices,

$$He = H\bar{V}e = \bar{P}e = e,$$

so H is also stochastic.

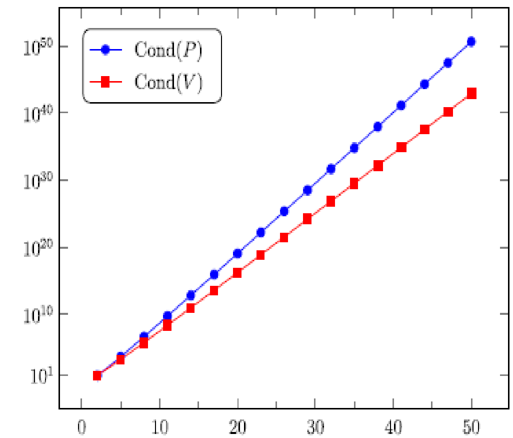

Fig. 1. Skeel condition numbers for Vandermonde and Pascal matrices versus the matrix size.

As a corollary to the stochastic nature of H, P, and V, we also have

$$\|\bar{P}\|_\infty = \|\bar{V}\|_\infty = \|H\|_\infty = 1.$$

Taking into mind that the Skeel condition number is invariant by row scaling, we can now derive (11) that P is nonnegative (thus |P| = P) and nonsingular.

$$\mathrm{Cond}(P) = \mathrm{Cond}(\bar{P}).$$

Since | P| = P and P is stochastic, we obtain by (14) that

$$\mathrm{Cond}(\bar{P}) = \| \, |\bar{P}^{-1}| \, |\bar{P}| \, \|_\infty = \| \, |\bar{P}^{-1}| \, |\bar{P}| e \|_\infty = \|\bar{P}^{-1}\|_\infty = \kappa_\infty(\bar{P}), \qquad (16)$$

$$\mathrm{Cond}(\bar{P}) = \| \, |\bar{P}^{-1}| \, |\bar{P}| \, \|_\infty = \| \, |\bar{P}^{-1}| \, |\bar{P}| e \|_\infty = \|\bar{P}^{-1}\|_\infty = \kappa_\infty(\bar{P}), \qquad (16)$$

where we have used the fact that $|\bar{P}|e = e$ and $\|\bar{P}\|_\infty = 1$.
From (15) and (16) we conclude that

$$\mathrm{Cond}(P) = \kappa_\infty(\bar{P}). \qquad (17)$$

That's what we can infer from this analogy

$$\mathrm{Cond}(V) = \kappa_\infty(\bar{V}). \qquad (18)$$

To put it in writing, use the rules of twelve and thirteen.

$$\bar{P}^{-1}H = \bar{V}^{-1}. \qquad (19)$$

We may get this conclusion from (14), (17), and (19).

$$\mathrm{Cond}(V) = \kappa_\infty(\bar{V}) = \|\bar{V}^{-1}\|_\infty = \|\bar{P}^{-1}H\|_\infty \leqslant \|\bar{P}^{-1}\|_\infty = \kappa_\infty(\bar{P}) = \mathrm{Cond}(P)$$

and the result follows. □

In spite of a Pascal matrix P's poor conditioning, one may nevertheless derive efficient algorithms by use of the matrix's bidiagonal decomposition BD (P). According to [8], we may discover HRA methods to carry out various calculations with a wholly nonnegative matrix A, such as the computation of a sub matrix of A from its bidiagonal factorization BD (A).

In terms of its singular values, eigenvalues, and inverse, or the solution of certain linear systems those in which a chessboard appears in b): Ax = b design characterized by a series of reversing indications). Given that Pascal matrices are bidiagonal, it is possible to develop very precise algorithms for them. For example, HRA is familiar with factorization. All the non-zero values in BD(P) are 1s.

Numerical results

Some numerical experiments are presented here. We begin by comparing Pascal and Vander monde matrices with respect to their Skeel condition numbers. Then, we demonstrate that it is still feasible to get precise numerical results using HRA despite the ill-conditioning by factoring in the bidiagonal decomposition of Pascal matrices using the techniques provided in [15,8]. In the form of Pascal matrices. As seen in Fig. 1, Vander monde and Pascal matrices up to size n = 50 have Skeel condition numbers shown by their respective colour bars. We can check that Theorem 1's inequality Cond(V) 6 Cond(P) holds.

Table 1
Relative errors of the eigenvalues of a 20 × 20 Pascal matrix.

λ	e_{conv}	e_{HRA}
$4.699483854180265 \cdot 10^{10}$	0	0
$5.549569598629316 \cdot 10^{8}$	$7.5183 \cdot 10^{-15}$	$4.2962 \cdot 10^{-16}$
$1.398271987686576 \cdot 10^{7}$	$1.1216 \cdot 10^{-13}$	$9.3247 \cdot 10^{-16}$
$5.667410592514715 \cdot 10^{5}$	$1.0349 \cdot 10^{-12}$	0
$3.306137385353127 \cdot 10^{4}$	$5.3936 \cdot 10^{-12}$	$8.8030 \cdot 10^{-16}$
$2.624520577381837 \cdot 10^{3}$	$2.0108 \cdot 10^{-11}$	$5.1981 \cdot 10^{-16}$
$2.753624643918711 \cdot 10^{2}$	$7.6078 \cdot 10^{-11}$	$2.0643 \cdot 10^{-16}$
$3.778382893288340 \cdot 10^{1}$	$3.2207 \cdot 10^{-10}$	0
$6.860716460896207 \cdot 10^{0}$	$1.1676 \cdot 10^{-9}$	$2.5892 \cdot 10^{-16}$
$1.748761542199869 \cdot 10^{0}$	$2.8461 \cdot 10^{-9}$	$7.6184 \cdot 10^{-16}$
$5.718332522008930 \cdot 10^{-1}$	$7.2625 \cdot 10^{-9}$	$7.7661 \cdot 10^{-16}$
$1.457573717992379 \cdot 10^{-1}$	$2.9716 \cdot 10^{-8}$	0
$2.646634891811339 \cdot 10^{-2}$	$1.4704 \cdot 10^{-7}$	$3.9327 \cdot 10^{-16}$
$3.631577027785784 \cdot 10^{-3}$	$9.0709 \cdot 10^{-7}$	$2.3884 \cdot 10^{-16}$
$3.810219697334504 \cdot 10^{-4}$	$6.7348 \cdot 10^{-6}$	$1.4228 \cdot 10^{-16}$
$3.024677693160022 \cdot 10^{-5}$	$5.3381 \cdot 10^{-5}$	$1.1202 \cdot 10^{-16}$
$1.764474240353715 \cdot 10^{-6}$	$3.8181 \cdot 10^{-4}$	$6.0006 \cdot 10^{-16}$
$7.151684427680540 \cdot 10^{-8}$	$1.6781 \cdot 10^{-3}$	$1.1104 \cdot 10^{-15}$
$1.801941542001724 \cdot 10^{-9}$	$1.1909 \cdot 10^{-2}$	$3.4429 \cdot 10^{-16}$
$2.127893256001899 \cdot 10^{-11}$	$6.3160 \cdot 10^{-1}$	$1.2148 \cdot 10^{-15}$

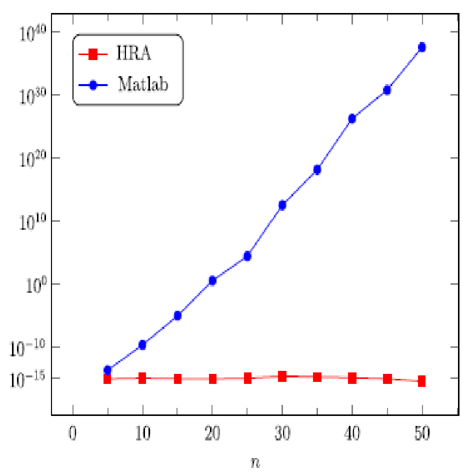

Fig. 2. Relative errors associated with the lowest eigenvalue of a Pascal matrix of order n with n = 5(5)50.

The eigenvalues of a Pascal matrix calculated using HRA and more standard techniques have been compared in a preliminary numerical test. The TNEigenValues procedure [15] was used to precisely calculate the eigenvalues, and the Matlab function eig was used for comparison. TNEigenValues's eigenvalues were put to the test through a comparison to those calculated by Mathematical with 60 decimal places of precision. Table 1 displays the eigenvalues (16 digits shown) and associated relative errors for a 20 by 20 Pascal matrix. Values of eigenvectors computed using HRA and the traditional technique. This is important to keep in mind since the relative inaccuracy of when using HRA; the eigenvalues are estimated to within a unit round off (in double precision) of what is obtained using the more traditional method increases in complexity as eigenvalue size decreases. In reality, the lowest eigenvalue shows the highest degree of inaccuracy.

I think everything is going swimmingly recognized that matrices with zero eigenvalues had zero eigenvalues overall (cf. [11]). In contrast, a bigger Pascal matrix dimensions (n > 20), the traditional algorithm's approximations to the weakest eigenvalues turn out to be a negative value indicates that this method should not be used to calculate eigenvalues of this kind. Also, keep in mind that, since a singular values and eigenvalues of a Pascal matrix of order n are equal, making the matrix symmetric. The relative error for the nth-order Pascal matrix's lowest eigenvalue is shown in Fig. 2. n = 5(5)50. While this demonstrates that HRA-related relative error is rather stable, it does not do so for error rates in other contexts. Traditional algorithms develop at an exponential rate. We have also solved linear systems of the to further demonstrate that precise answers are attainable using Pascal matrices. Px = b, where b is a variable with a changing sign. The system solution is available to HRA in this case. And this at the TNSolve procedure [16]. The typical relative errors of the approximations are shown in Table 2.

Table 2
Average relative errors for the approximate solution of a linear
System of the form Px = b.

n	$\langle e_{conv} \rangle$	$\langle e_{HRA} \rangle$
5	$2.4292 \cdot 10^{-16}$	$9.5635 \cdot 10^{-17}$
10	$7.0317 \cdot 10^{-15}$	$1.1479 \cdot 10^{-16}$
15	$2.7774 \cdot 10^{-13}$	$1.3792 \cdot 10^{-16}$
20	$5.8856 \cdot 10^{-12}$	$1.3644 \cdot 10^{-16}$
25	$2.8650 \cdot 10^{-10}$	$1.8974 \cdot 10^{-16}$

Table 3
Maximum relative error of the elements of P−1 computed to HRA and with a conventional algorithm.

n	e_{conv}	e_{HRA}
5	$1.3989 \cdot 10^{-14}$	$1.7764 \cdot 10^{-16}$
10	$4.7017 \cdot 10^{-10}$	$5.8003 \cdot 10^{-16}$
15	$8.4625 \cdot 10^{-06}$	$1.1102 \cdot 10^{-15}$
20	$2.1371 \cdot 10^{+00}$	$1.1219 \cdot 10^{-15}$
25	$1.0402 \cdot 10^{+00}$	$2.3007 \cdot 10^{-15}$

Linear systems and the like. The findings were averaged across 20 measurements and the errors were calculated using the euclidean \snorm. Assuming that b is a uniformly distributed random integer between 0 and 1, we generate the elements of b1:= |b| to be uniformly distributed random numbers between 0 and 1, and we set b to be b(i) = (1)ib1 (i). Mathematical was used to get the precise answers to the systems. As predicted, the error is less with HRA calculations, and this becomes increasingly apparent as the size of the system increases. We have completed the Pascal matrix HRA inverse computation. The TNJinverse method [16] allowed this to be accomplished. Matlab's inv function was selected as the industry standard algorithm. Invers matrices were calculated exactly by using Mathematical. Maximum relative errors of P1 elements up to size n = 25 are shown in Table 3.

Conclusions

Pascal matrices are shown to always be worse conditioned for the Skeel condition number than Vander monde matrices of the same order, showing their ill-conditioning. Numerical tests, however, have revealed that there exist procedures that enable us to precisely determine the inverses and eigenvalues of these matrices. Particular linear system solving

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