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E-Mail :
editor.ijasem@gmail.com
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MARSHALL-OLKIN MODIFIED EXPONENTIATED HALF LOGISTIC DISTRIBUTION – PROPERTIES AND ESTIMATION

Mayuri Odela, M.Sc.(Mathematics),B.Ed *1, Y. Ananda Reddy M.Sc.(Statistics), M.Sc. (Mathematics) *2, G. Venkateshwarlu, MCA, M.Tech. *3

ABSTRACT

In this paper, we proposed a Modified Exponentiated Half Logistic Distribution (MOEHLD) with some properties. The method of Maximum Likelihood Estimation was used to estimate and evaluate the unknown parameters of the MOEHLD illustrated by a life time data set.

Key words: Reliability, Hazard Function, Maximum Likelihood Estimation.

1. INTRODUCTION

Statistical distribution and dissemination play a vital role in analyzing and assessing the authentic scenario of the real world. Indeed, the fact that moderate number of distributions has been developed. There is always scope to developing distributions, analyzing their properties that are more flexible or to adjust real world scenarios. The researchers are continually urging for establishing new and more flexible distributions. For that reason, many new distributions have been emerged and studies. In current works, new distributions are outlined by means of including one or more parameters to a distribution functions. Such an addition of parameters makes the ensuing distribution richer for modelling life time data. The generalized distributions have been invented to characterize different phenomena. These generalized distributions are also having a greater number of parameters. Johnson *et al.* (1994) clearly emphasized the four parameter distributions that are much essential for the workable situations. Many authors were not having crystal-clear opinion on whether three parameters or more than three were necessary for a better analysis. Adding too many parameters to the distribution may not help for a successful inference. "Proportional hazard model (PHM), Proportional reversed hazard model (PRHM), Proportional odds model (POM), Power transformed model (PTM), are few such models originated from this idea to add a shape parameter". In recent years, many of the distributions have been developed based on the beta distribution. The cumulative distribution function (cdf) of generalized beta distribution for the random variable (X) is defined by,

$$F(x) = \frac{1}{B(\alpha,\beta)} \int_0^{G(x)} t^{\alpha-1}(1-t)^{\beta-1}\,dt;\ t>0,\alpha,\beta>0$$

*1. Faculty, Department of Mathematics, Siva Sivani Degree College – Kompally – 500 100.
*2. H.O.D, Department of Statistics, Siva Sivani Degree College – Kompally – 500 100.
*3. Faculty, Department of Computer Science, Siva Sivani Degree College–Kompally–100.

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where G(x) is the cdf of any other distribution. The above function is an example of invention of new distribution by addition of parameter(s). The following authors who studied the above class of distributions are "Eugene *et al.* (2002), Nadarajah and Kotz (2004, 2006), Famoye *et al.* (2005), Kozubowski and Nadarajah (2008), Akinsete *et al.* (2008), Akinsete and Lowe (2009)". By studying all these articles, we decided to develop a new generalized distribution.

Introducing a flexible distribution by adding a parameter that acts as both a shape parameter and a scale parameter ensures accuracy when fitting datasets in medicine, reliability engineering, and finance. This study has mainly focused on the shape or scale parameters which can accurately determine the new family distributions. The level of flexibility may increase due to introduce both parameters shape and scale.

For generalizing the existing probability distributions, a new family distribution is introduced by adding scale, shape and location parameters. One of the most interesting ways to add shape parameters to an existing distribution is exponentiation. Augmented family precursors from Mudholkar and Srivastava (1993) are defined by the following cdf.

$$G\left(x;c,\xi\right) = F\left(x;\xi\right)^{c}, c,\xi > 0, x \in R,$$

where c is considered as the additional shape parameter.

The prominent authors like Marshall and Olkin initiated a novel introducing a single-scale parameter to a family distribution. Marshall-Olkin's (MO) (1997) cdf of family is as follows

$$G\left(x,\sigma,\xi\right) = \frac{F\left(x;\xi\right)}{\sigma+\left(1-\sigma\right)F\left(x;\xi\right)}, \sigma,\xi > 0, x \in R,$$

where σ is taken as an extra parameter.

Undeniably the scale and shape parameters increase the degree of flexibility in the family distribution, but on the other hand the huge increase in the parameters, may also affect the calculation of mathematical functions making things more complex and complicated.

There are various research applications of EHLD in real time phenomenon. El-sayed El-Sherpieny and Mahmoud Elsehetry (2019) developed Kumaraswamy Type I half logistic family of distributions with applications. Amal Hassan *et al.* (2017) developed type II half logistic family of distributions with applications. Manisha Pal *et al.* (2003) established exponentiated weibull distribution. Gupta and Kundu (2001) studied exponentiated exponential family: An alternative to gamma and weibull distributions. The reliability of the system is the possibility that the system can fully perform its intended purpose within a given time under specified environmental conditions. Kapur *et al.* (1997) studied the reliability in engineering design. The reliability of the multi component stress strength model established by Bhattacharyya and Johnson (1974).

2. Marshall-Olkin Modified Exponentiated Half Logistic Distribution

Here we introduced the Modified Exponentiated Half Logistic Distribution (MOEHLD). By considering the Exponentiated Half-Logistic distribution with parameter

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$\beta,\delta>0$, the cdf and probability density function (pdf) of Exponentitated Half Logistic distribution are given by

$$F(x;\beta,\delta)=\left[\frac{1-\exp(-x/\delta)}{1+\exp(-x/\delta)}\right]^{\beta} \quad x\geq 0,\beta,\delta>o \tag{1}$$

And

$$f(x;\beta,\delta) = \frac{2\beta\exp(-x/\delta)\left(1-\exp\left(-x/\delta\right)\right)^{\beta-1}}{\delta\left(1+\exp\left(-x/\delta\right)\right)^{\beta+1}};x\geq 0\ \beta,\delta>0 \tag{2}$$

The cdf of MOEHLD is given by

$$G(x;\lambda,\beta,\delta)=1-\frac{\lambda\left[1-\left[\frac{1-\exp(-x/\delta)}{1+\exp(-x/\delta)}\right]^{\beta}\right]}{\left[1-(1-\lambda)\left(1-\left[\frac{1-\exp(-x/\delta)}{1+\exp(-x/\delta)}\right]^{\beta}\right)\right]}$$

$$=\frac{\left(1-\exp(-x/\delta)\right)^{\beta}}{\left[\lambda\left(1+\exp(-x/\delta)\right)^{\beta}+(1-\lambda)\left(1-\exp(-x/\delta)\right)^{\beta}\right]} \quad x>0,\ \beta,\delta,\lambda>0 \tag{3}$$

The pdf of MOEHLD is given by

$$g(x;\lambda,\beta,\delta)=\frac{\partial}{\partial x}\left[G\left(x;\lambda,\beta,\delta\right)\right]$$

$$=\frac{2\beta\lambda\exp(-x/\delta)(1-\exp(-2x/\delta))^{\beta-1}}{\delta\left[\lambda(1+\exp(-x/\delta))^{\beta}+\left(1-\lambda\right)(1-\exp(-x/\delta))^{\beta}\right]^{2}} \quad x>0,\ \beta,\delta,\lambda>0 \tag{4}$$

Reliability function of MOEHLD is given by

$$R\left(x\right)=1-G\left(x\right)$$

$$=\frac{\lambda\left[\left(1+\exp(-x/\delta)\right)^{\beta}-\left(1-\exp(-x/\delta)\right)^{\beta}\right]}{\lambda\left(1+\exp(-x/\delta)\right)^{\beta}+(1-\lambda)\left(1-\exp(-x/\delta)\right)^{\beta}} \tag{5}$$

Hazard function is given by

$$h\left(x\right)=\frac{g\left(x\right)}{R\left(x\right)}$$

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$$= \frac{2\beta\lambda \exp(-x/\delta)(1-\exp(-2x/\delta)^{\beta-1}}{\delta\left[\lambda(1+\exp(-x/\delta))^{\beta}+(1-\lambda)(1-\exp(-x/\delta))^{\beta}\right]\left[(1+\exp(-x/\delta))^{\beta}-(1-\exp(-x/\delta))^{\beta}\right]} \quad (6)$$

With different combination values of the parameters shapes of the distribution, density, reliability and hazard functions of the MOEHLD are given from Figure 1 to Figure 4 respectively.

GRAPHICAL REPRESENTATION OF CDF, PDF, RELIABILITY AND HAZARD FUNCTION

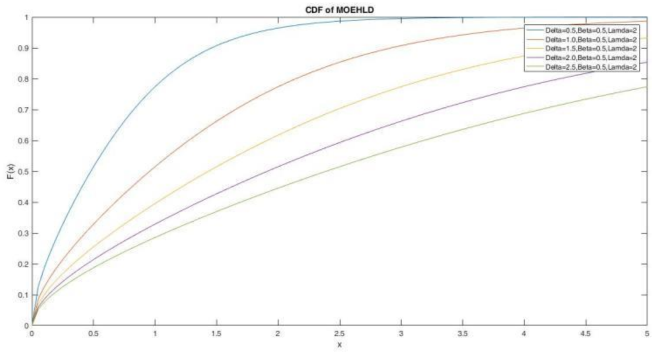

Fig:1 Cumulative Distribution Function

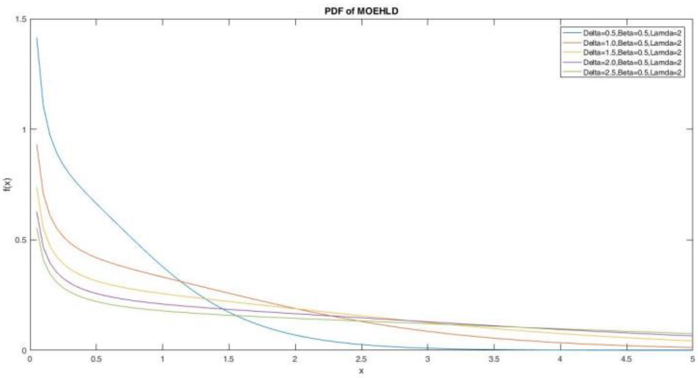

Fig:2 Probability Distribution Function

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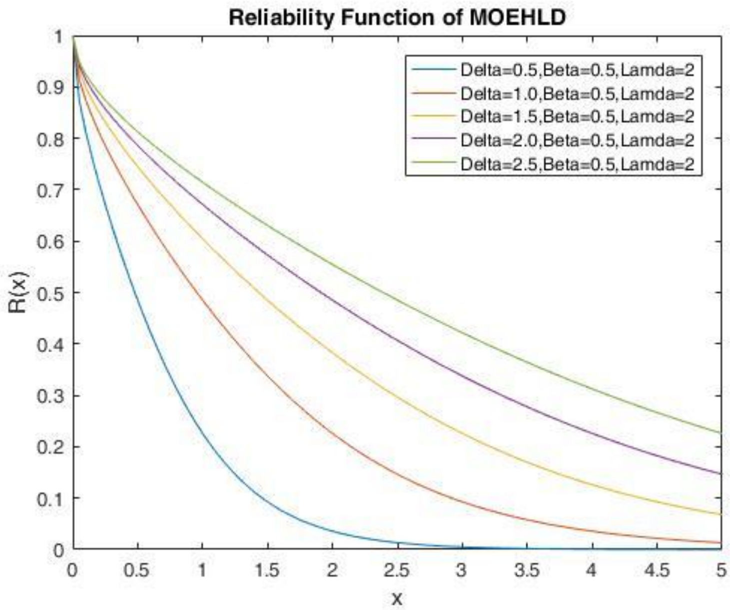

Fig:3 Reliability Function

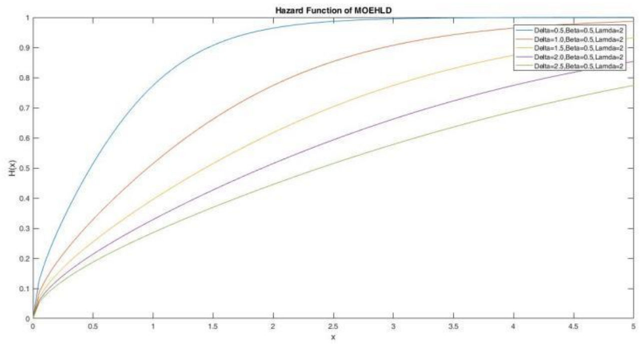

Fig:4 Hazard Function

3. PROPERTIES OF THE MOEHLD

Quantile Function:

$$x_q = -\delta \log_e \left[1 - \frac{2}{\left(1 + \left\{ \frac{(1/q) - (1-\lambda)}{\lambda} \right\}^{1/\beta} \right)} \right]$$

4. Maximum Likelihood Estimation

The most preferred method to estimate and evaluate the unknown and unidentified parameters is the Maximum Likelihood Estimation which is shortly known as MLE. This method is extensively used as it is asymptotically consistent, unbiased and normal as the sample size tends to large. The MLE, to estimate the parameters of certain distributions does not yield

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explicit solutions from complete/censored samples. However, the MLE method is currently one of the most used methods of estimation, for its versatile and provides reliable results. These parameters are obtained by maximizing the likelihood function of the model under consideration.

By using the MLE method, we can estimate the parameters of MOEHLD. The likelihood and log likelihood functions are given by

$L = \prod_{i=1}^{n} g\left(x_i, \beta, \delta, \lambda\right)$ and

$$log\ \mathrm{L} = \log\left[\prod_{i=1}^{n} g\left(x_i, \beta, \delta, \lambda\right)\right]$$

$$\log L = \log\left(\prod_{\mathrm{i=1}}^{\mathrm{n}}\left[\frac{2\beta\lambda\exp(-x/\delta)(1-\exp(-2x/\delta))^{\beta-1}}{\delta\left[\lambda(1+\exp(-x/\delta))^{\beta}+\left(1-\lambda\right)(1-\exp(-x/\delta))^{\beta}\right]^{2}}\right]\right)$$

We can obtain the Maximum Likelihood estimates of the parameters β, δ and λ by solving the first derivatives of the equations.

$$\frac{\partial \log L}{\partial \lambda} = 0$$

$$\Rightarrow \frac{n}{\lambda} - 2\sum_{i=1}^{n}\frac{A_i^{\beta} - B_i^{\beta}}{\lambda A_i^{\beta} + (1-\lambda)B_i^{\beta}} = 0 \tag{7}$$

$$\frac{\partial \log L}{\partial \beta} = 0$$

$$\Rightarrow \frac{n}{\beta} + \sum_{i=1}^{n}\log C_i^{\beta} - 2\sum_{i=1}^{n}\frac{\lambda A_i^{\beta}\log A_i + (1-\lambda)B_i^{\beta}\log B_i}{\left[\lambda A_i^{\beta} + (1-\lambda)B_i^{\beta}\right]} = 0 \tag{8}$$

$$\frac{\partial \log L}{\partial \delta} = 0$$

$$\Rightarrow \frac{\sum_{i=1}^{n} x_i}{\delta^2} - 2(\beta-1)\sum_{i=1}^{n}\frac{x_i\exp(-2x_i/\delta)}{\delta^2 C_i} - \frac{n}{\delta} - 2\sum_{i=1}^{n}\frac{\beta x_i\exp(x_i/\delta)\left[\lambda A_i^{\beta-1} - (1-\lambda)B_i^{\beta-1}\right]}{\delta^2\left[\lambda A_i^{\beta} + (1-\lambda)B_i^{\beta}\right]} = 0 \tag{9}$$

The second order derivatives of the log likelihood function of MOEHLD with respect to the parameter λ, β and δ

$$\frac{\partial^2 \log L}{\partial \lambda^2} = \frac{\partial}{\partial \lambda}\left[\frac{n}{\lambda} - 2\sum_{i=1}^{n} \frac{A_i^\beta - B_i^\beta}{\lambda A_i^\beta + (1-\lambda)B_i^\beta}\right]$$

$$= -\frac{n}{\lambda^2} + 2\sum_{i=1}^{n}\left[\frac{\left(A_i^\beta - B_i^\beta\right)^2}{\left(\lambda A_i^\beta + (1-\lambda)B_i^\beta\right)^2}\right] \quad (10)$$

$$\frac{\partial^2 \log L}{\partial \beta^2} = \frac{\partial}{\partial \beta}\left[\frac{n}{\beta} + \sum_{i=1}^{n} \log C_i^\beta - 2\sum_{i=1}^{n} \frac{\lambda A_i^\beta \log A_i + (1-\lambda)B_i^\beta \log B_i}{\left[\lambda A_i^\beta + (1-\lambda)B_i^\beta\right]}\right]$$

$$= -\frac{n}{\beta^2} - 2\sum_{i=1}^{n}\left[\frac{\lambda(1-\lambda)C_i^\beta \left(\log A_i^\beta - \log B_i^\beta\right)^2}{\left(\lambda A_i^\beta + (1-\lambda)B_i^\beta\right)^2}\right] \quad (11)$$

$$\frac{\partial^2 \log L}{\partial \lambda \partial \beta} = \frac{\partial}{\partial \beta}\left[\frac{n}{\lambda} - 2\sum_{i=1}^{n} \frac{A_i^\beta - B_i^\beta}{\lambda A_i^\beta + (1-\lambda)B_i^\beta}\right]$$

$$= -2\sum_{i=1}^{n} \frac{C_i^\beta \log(A_i / B_i)}{\left[\lambda A_i^\beta + (1-\lambda)B_i^\beta\right]^2} \quad (12)$$

$$= \frac{\partial^2 \log L}{\partial \beta \partial \lambda}$$

$$\frac{\partial^2 \log L}{\partial \lambda \partial \delta} = \frac{\partial}{\partial \delta}\left[\frac{n}{\lambda} - 2\sum_{i=1}^{n} \frac{A_i^\beta - B_i^\beta}{\lambda A_i^\beta + (1-\lambda)B_i^\beta}\right]$$

$$= -2\sum_{i=1}^{n} \frac{\beta x_i \exp(-x_i/\delta)\left[A_i^\beta B_i^{\beta-1} + A_i^{\beta-1} B_i^\beta\right]}{\delta^2 \left[\lambda A_i^\beta + (1-\lambda)B_i^\beta\right]^2} \quad (13)$$

$$= \frac{\partial^2 \log L}{\partial \delta \partial \lambda}$$

$$\frac{\partial^2 \log L}{\partial \beta \partial \delta} = \frac{\partial}{\partial \delta}\left[\frac{n}{\beta} + \sum_{i=1}^{n} \log C_i^\beta - 2\sum_{i=1}^{n} \frac{\lambda A_i^\beta \log A_i + (1-\lambda)B_i^\beta \log B_i}{\left[\lambda A_i^\beta + (1-\lambda)B_i^\beta\right]}\right]$$

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$$= -2\sum_{i=1}^{n} \frac{x_i \exp(-2x_i/\delta)}{\delta^2 C_i} - 2\sum_{i=1}^{n} \frac{\exp(-x_i/\delta)x_i\left[\lambda^2 A_i^{2\beta-1} - (1-\lambda)^2 B_i^{2\beta-1} + \lambda(1-\lambda)\left(A_i^{\beta-1}B_i^{\beta} - A_i^{\beta}B_i^{\beta-1}\right) + \lambda\beta\log(A_i/B_i)\left(A_i^{\beta-1}B_i^{\beta} + A_i^{\beta}B_i^{\beta-1}\right)\right]}{\delta^2\left[\left[\lambda A_i^{\beta} + (1-\lambda)B_i^{\beta}\right]^2\right]}$$

(14)

$$= \frac{\partial^2 \log L}{\partial\delta\partial\beta}$$

Where $A_i = 1 + e^{\frac{-x_i}{\delta}}$, $B_i = 1 - e^{\frac{-x_i}{\delta}}$ and $C_i = 1 - e^{\frac{-2x_i}{\delta}}$

Asymptotic Confidence Interval

Here an attempt has been made to derive the asymptotic confidence intervals of the unknown parameters δ, β and λ . Using large sample approach and assume that the distribution of MLE's of (δ, λ, β) which are approximately multivariate normal with mean (δ, β, λ) and variance-covariance matrix I^{-1} , where I^{-1} is observe information matrix which is given as

$$I^{-1} = \begin{bmatrix} \frac{\partial^2 \log L}{\partial\delta^2} & \frac{\partial^2 \log L}{\partial\delta\partial\beta} & \frac{\partial^2 \log L}{\partial\delta\partial\lambda} \\ \frac{\partial^2 \log L}{\partial\beta\partial\delta} & \frac{\partial^2 \log L}{\partial\beta^2} & \frac{\partial^2 \log L}{\partial\beta\partial\lambda} \\ \frac{\partial^2 \log L}{\partial\lambda\partial\delta} & \frac{\partial^2 \log L}{\partial\lambda\partial\beta} & \frac{\partial^2 \log L}{\partial\lambda^2} \end{bmatrix}^{-1}_{\delta=\delta,\beta=\beta,\lambda=\lambda}$$

$$= \begin{bmatrix} \mathrm{var}(\delta) & \mathrm{cov}(\delta,\beta) & \mathrm{cov}(\delta,\lambda) \\ \mathrm{cov}(\beta,\delta) & \mathrm{var}(\beta) & \mathrm{cov}(\beta,\lambda) \\ \mathrm{cov}(\lambda,\delta) & \mathrm{cov}(\lambda,\beta) & \mathrm{var}(\lambda) \end{bmatrix}$$

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