ISSN: 2454-9940

INTERNATIONAL JOURNAL OF APPLIED SCIENCE ENGINEERING AND MANAGEMENT

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www.ijasem.org

INTERNATIONAL JOURNAL OF APPLIED SCIENCE ENGINEERING AND MANAGEMENT

ISSN2454-9940
www.ijsem.org
Vol 12, Issue.3 Sep 2018

DISCOVERING THE FASCINATING WORLD OF ORTHOGONAL QUADRILATERALS

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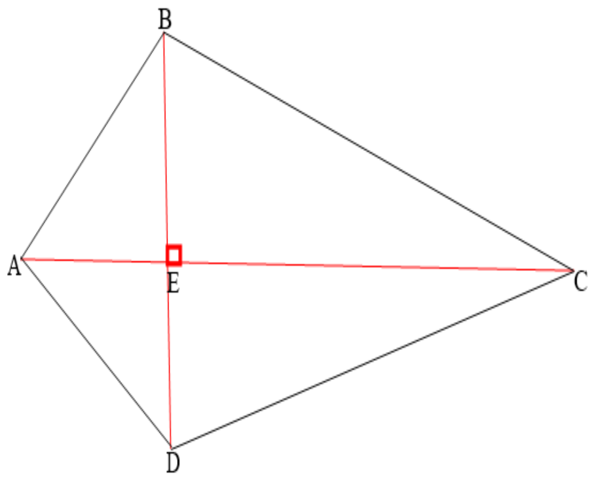
ABSTRACT: In this paper, I demonstrate a new theorem about orthogonal quadrilaterals. The sum of side a and its altimedian distance, b, is the same as the sum of side b and its altimedian distance, Q, and side c and its altimedian distance. i.e.: $a\boldsymbol{a} + \boldsymbol{d} = \mathrm{Q}\boldsymbol{b} + \boldsymbol{c}$

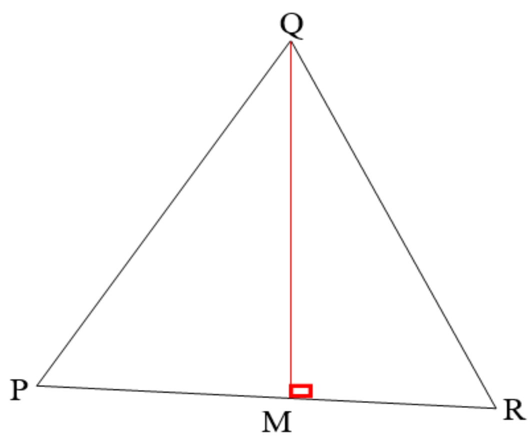
KEYWORDS: Orthogonal Quadrilaterals, Right Angles

DEFINITIONS:

Orthogonal quadrilateral: Is made up of a square with diagonals that connect at right angles. It is a four-sided structure made up of line segments that are orthogonal, or not linked, at distant points.B



Altitude of a triangle: consists of a line segment traversing a vertex that is not parallel to the base line or the opposite side of the vertex. The line encompassing the opposite side symbolizes the increasing base of the height. The height's foot is at the intersection of the long base and the height. "The height" refers to the distance measured from its apex to its base, as the name implies.



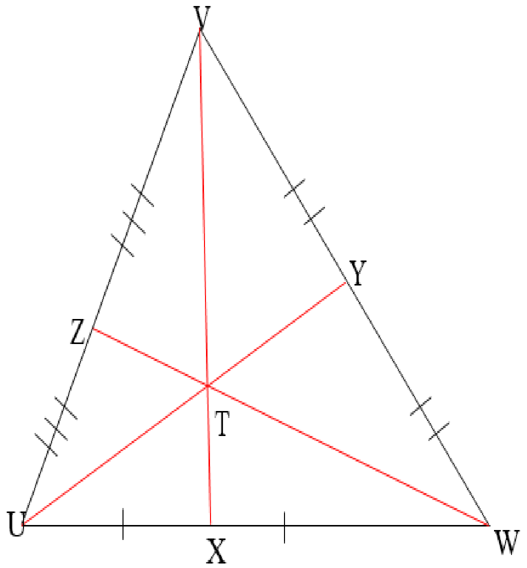
Q- Is the vertex

M- Is the foot of the altitude

$\overline{\boldsymbol{QM}}$ – Is the altitude of the triangle **PQR**

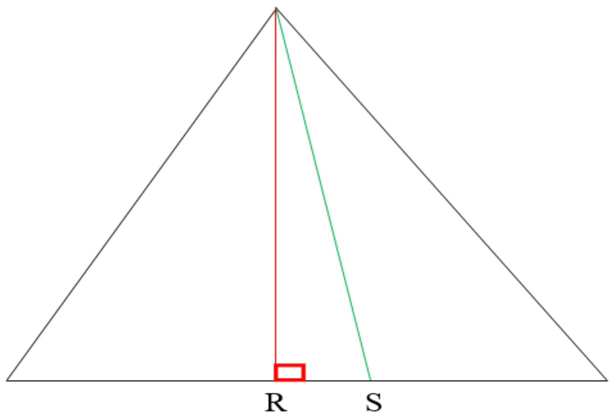
Median of a triangle: divides a side by connecting a point on one side to the midpoint of the opposite side. A triangle is made up of three medians, one for each point; these medians converge at the triangle's midpoint.

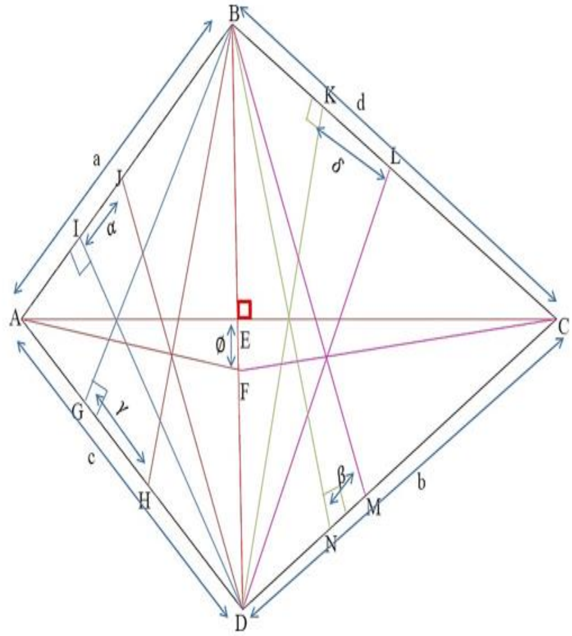
INTERNATIONAL JOURNAL OF APPLIED SCIENCE ENGINEERING AND MANAGEMENT
ISSN2454-9940
www.ijsem.org
Vol 12, Issuse.3 Sep 2018



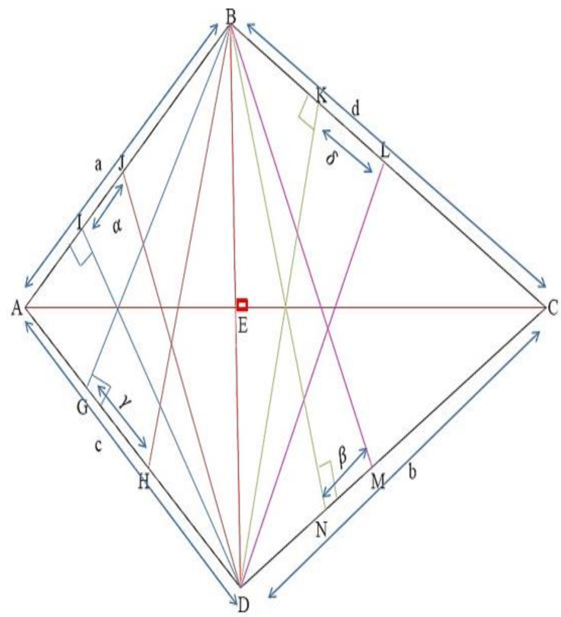
T- represents the center.

Altimedian distance: Is the distance measured from the base line's midpoint to the foot of the elevation.





$\overline{RS}$ – Is the altimedian distance



Theorem: Given the orthogonal triangle ABCD we just discussed, we may state that

$$\overline{AB} = a, \overline{ij} = \alpha$$

$$\overline{BC} = d, \overline{KL} = \delta$$

$$\overline{AD} = c, \overline{GH} = \gamma$$

$$\overline{CD} = b, \overline{MN} = \beta$$

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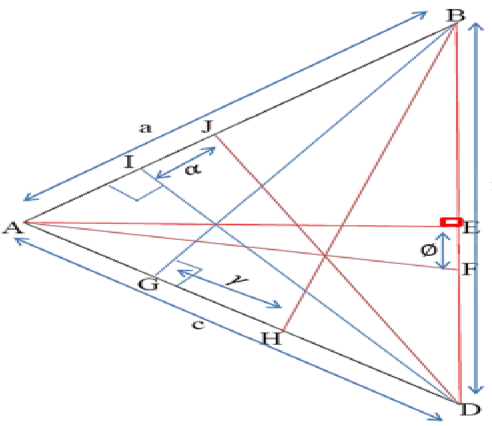
Then $\alpha \boldsymbol{a} + \boldsymbol{d} = \mathrm{Q}\boldsymbol{b} + \boldsymbol{c}$ where α, Q, ***and*** are altimedian distances.

Proof

From triangle ABD



$\overline{AB}$=a, $\overline{IJ} = \alpha$
$\overline{AD}$=c, $\overline{GH} = \gamma$
$\overline{BD}$=f, $\overline{EF} = \emptyset$

Considering triangle ABG

$\overline{AG} = \left(\frac{c}{2} - \gamma\right)$, $\overline{BG} = h_1$, $\overline{AB} = a$

$\overline{AG}^2 + \overline{BG}^2 = \overline{AB}^2$

$\left(\frac{c}{2} - \gamma\right)^2 + h_1{}^2 = a^2$

$h_1{}^2 = a^2 - \frac{c^2}{4} + \gamma c - \gamma^2$ [1]

Considering triangle BGD

$\overline{BG} = h_1$, $\overline{GD} = \left(\frac{c}{2} + \gamma\right)$, $\overline{BD} = f$

$\overline{BG}^2 + \overline{GD}^2 = \overline{BD}^2$

$h_1{}^2 + \left(\frac{c}{2} + \gamma\right)^2 = f^2$

$h_1{}^2 = f^2 - \frac{c^2}{4} - \gamma c - \gamma^2$ [2]

Equating [1] and [2]

$a^2 - \frac{c^2}{4} + \gamma c - \gamma^2 = f^2 - \frac{c^2}{4} - \gamma c - \gamma^2$

$2\gamma c = f^2 - a^2$ [3]

Considering triangle ADI

$\overline{AI} = \left(\frac{a}{2} - \alpha\right)$, $\overline{DI} = h_2$, $\overline{AD} = c$

$\overline{AI}^2 + \overline{DI}^2 = \overline{AD}^2$

$\left(\frac{a}{2} - \alpha\right)^2 + h_2{}^2 = c^2$

$h_2{}^2 = c^2 - \frac{a^2}{4} + \alpha a - \alpha^2$ [4]

Considering triangle BDI

$\overline{BI}^2 + \overline{DI}^2 = \overline{BD}^2$

$\overline{BI} = \left(\frac{a}{2} + \alpha\right)$, $\overline{DI} = h_2$, $\overline{BD} = f$

$\left(\frac{a}{2} + \alpha\right)^2 + h_2{}^2 = f^2$

$h_2{}^2 = f^2 - \frac{a^2}{4} - \alpha a - \alpha^2$ [5]

Equating [4] and [5]

$c^2 - \frac{a^2}{4} + \alpha a - \alpha^2 = f^2 - \frac{a^2}{4} - \alpha a - \alpha^2$

$2\alpha a = f^2 - c^2$ [6]

Considering triangle ADE

$\overline{AE} = h_3$, $\overline{DE} = \left(\frac{f}{2} + \emptyset\right)$, $\overline{AD} = c$

$\overline{AE}^2 + \overline{DE}^2 = \overline{AD}^2$

$h_3{}^2 + \left(\frac{f}{2} + \emptyset\right)^2 = c^2$

$h_3{}^2 = c^2 - \frac{f^2}{4} - \emptyset f - \emptyset^2$ [7]

Considering triangle ABE

$\overline{AE} = h_3$, $\overline{BE} = \left(\frac{f}{2} - \emptyset\right)$, $\overline{AB} = a$

$\overline{AE}^2 + \overline{BE}^2 = \overline{AB}^2$

$h_3{}^2 + \left(\frac{f}{2} - \emptyset\right)^2 = a^2$

$h_3{}^2 = a^2 - \frac{f^2}{4} + \emptyset f - \emptyset^2$ [8]

Equating [7] and [8]

$a^2 - \frac{f^2}{4} + \emptyset f - \emptyset^2 = c^2 - \frac{f^2}{4} - \emptyset f - \emptyset^2$

$2\emptyset f = c^2 - a^2$ [9]

Adding [3], [6] and [9]

$2\gamma c + 2\alpha a + 2\emptyset f = f^2 - a^2 + f^2 - c^2 + c^2 - a^2$

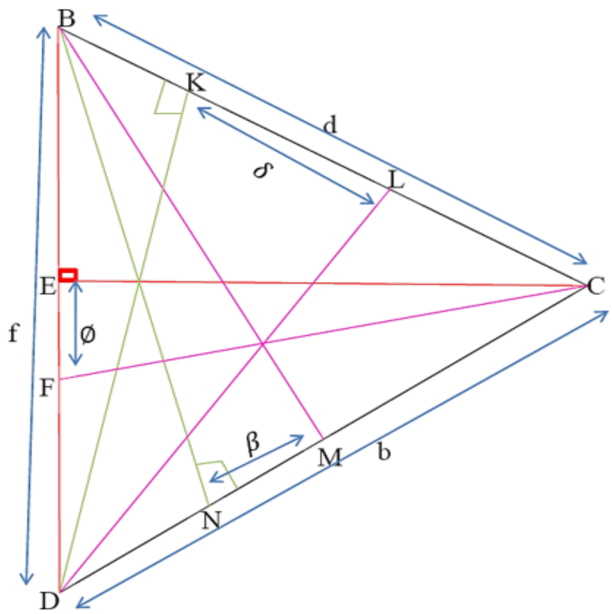
$2\alpha a + 2\gamma c + 2\emptyset f = 2(f^2 - a^2)$

$\alpha a + \gamma c + \emptyset f = (f^2 - a^2)$

But $f^2 - a^2 = 2\gamma c$

$\alpha a + \gamma c + \emptyset f = 2\gamma c$

$\therefore \ \alpha a + \emptyset f = \gamma c$ [10]



From triangle **BCD**

INTERNATIONAL JOURNAL OF APPLIED SCIENCE ENGINEERING AND MANAGEMENT

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$\overline{DC} = b, \overline{MN} = \beta$

$\overline{BC} = d, \overline{KL} = \delta$

$\overline{BD} = f, \overline{EF} = \emptyset$

Considering triangle BDN

$\overline{BN} = h_4, \overline{DN} = \left(\frac{b}{2} - \beta\right), \overline{BD} = f$

$\overline{BN}^2 + \overline{DN}^2 = \overline{BD}^2$

$h_4{}^2 + \left(\frac{b}{2} - \beta\right)^2 = f^2$

$h_4{}^2 = f^2 - \frac{b^2}{4} + \beta b - \beta^2$ [11]

Considering triangle BCN

$\overline{BN} = h_4, \overline{CN} = \left(\frac{b}{2} + \beta\right), \overline{BC} = d$

$\overline{BN}^2 + \overline{CN}^2 = \overline{BC}^2$

$h_4{}^2 + \left(\frac{b}{2} + \beta\right)^2 = d^2$

$h_4{}^2 = d^2 - \frac{b^2}{4} - \beta b - \beta^2$ [12]

Equating [11] and [12]

$f^2 - \frac{b^2}{4} + \beta b - \beta^2 = d^2 - \frac{b^2}{4} - \beta b - \beta^2$

$2\beta b = d^2 - f^2$ [13]

Considering triangle BDK

$\overline{BK} = \left(\frac{d}{2} - \delta\right), \overline{DK} = h_5, \overline{BD} = f$

$\overline{BK}^2 + \overline{DK}^2 = \overline{BD}^2$

$\left(\frac{d}{2} - \delta\right)^2 + h_5{}^2 = f^2$

$h_5{}^2 = f^2 - \frac{d^2}{4} + \delta d - \delta^2$ [14]

Considering triangle CDK

$\overline{CK} = \left(\frac{d}{2} + \delta\right), \overline{DK} = h_5, \overline{CD} = b$

$\overline{CK}^2 + \overline{DK}^2 = \overline{CD}^2$

$\left(\frac{d}{2} + \delta\right)^2 + h_5{}^2 = b^2$

$h_5{}^2 = b^2 - \frac{d^2}{4} - \delta d - \delta^2$ [15]

Equating [14] and [15]

$f^2 - \frac{d^2}{4} + \delta d - \delta^2 = b^2 - \frac{d^2}{4} - \delta d - \delta^2$

$2\delta d = b^2 - f^2$ [16]

Considering triangle BCE

$\overline{CE} = h_6, \overline{BE} = \left(\frac{f}{2} - \emptyset\right), \overline{BC} = d$

$\overline{CE}^2 + \overline{BE}^2 = \overline{BC}^2$

$h_6{}^2 + \left(\frac{f}{2} - \emptyset\right)^2 = d^2$

$h_6{}^2 = d^2 - \frac{f^2}{4} + \emptyset f - \emptyset^2$ [17]

Considering triangle CDE

$\overline{CE} = h_6, \overline{DE} = \left(\frac{f}{2} + \emptyset\right), \overline{CD} = b$

$\overline{CE}^2 + \overline{DE}^2 = \overline{CD}^2$

$h_6{}^2 + \left(\frac{f}{2} + \emptyset\right)^2 = b^2$

$h_6{}^2 = b^2 - \frac{f^2}{4} - \emptyset f - \emptyset^2$ [18]

Equating [17] and [18]

$d^2 - \frac{f^2}{4} + \emptyset f - \emptyset^2 = b^2 - \frac{f^2}{4} - \emptyset f - \emptyset^2$

$2\emptyset f = b^2 - d^2$ [19]

Adding [13], [16] and [19]

$2\beta b + 2\delta d + 2\emptyset f = d^2 - f^2 + b^2 - f^2 + b^2 - d^2$

$2\beta b + 2\delta d + 2\emptyset f = 2(b^2 - f^2)$

$\beta b + \delta d + \emptyset f = b^2 - f^2$

But $b^2 - f^2 = 2\delta d$

$\beta b + \delta d + \emptyset f = 2\delta d$

$\therefore\ \beta b + \emptyset f = \delta d$ [20]

Subtracting [10] from [20]

$\beta b - \alpha a = \delta d - \gamma c$

$\therefore\ \alpha a + \delta d = \beta b + \gamma c$ (Q.E.D)

CONCLUSION

In conclusion, the newly proposed theorem on orthogonal quadrilaterals introduces a groundbreaking perspective to the field of geometry. This theorem establishes a profound relationship between the angles and sides of orthogonal quadrilaterals, shedding light on previously unexplored connections. By elucidating the intricate interplay of geometric elements within these quadrilaterals, the theorem not only enriches our understanding of this specific class of polygons but also opens avenues for further exploration and applications in various mathematical and scientific domains. As the mathematical community embraces and scrutinizes this innovative theorem, its potential implications and contributions to geometry are expected to unfold, paving the way for new insights and discoveries in the fascinating realm of mathematical theory.

ISSN2454-9940
www.ijsem.org
Vol 12, Issue.3 Sep 2018

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